

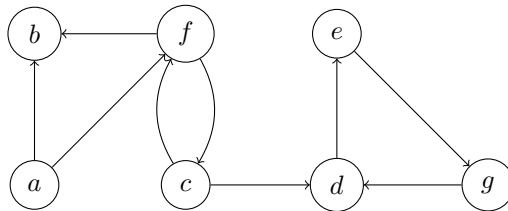
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## DSC 40B - Discussion 07

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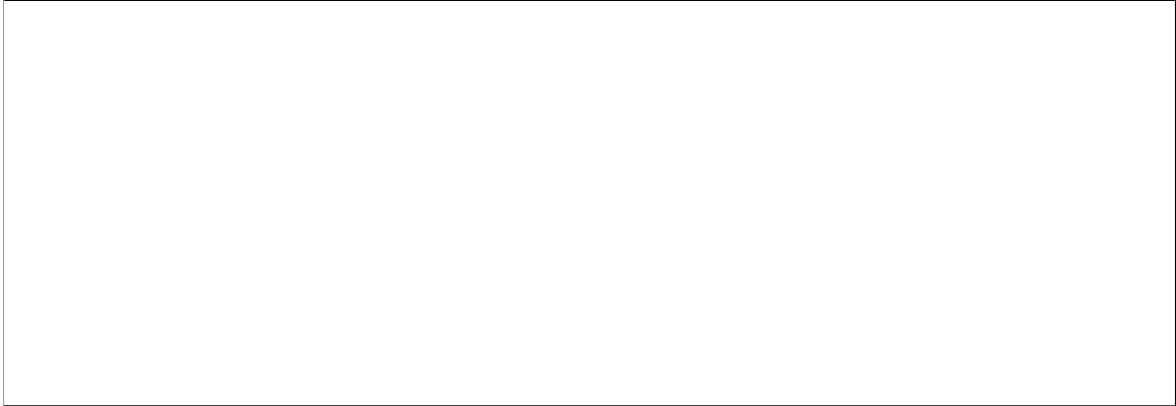
### Problem 1.

Consider a *breadth*-first search on the graph shown in the figure, starting with node *c*.



- a) Suppose you call `bfs_shortest_paths(graph, 'c')` on the graph above. This function returns dictionaries `distance` and `predecessor`. Write down the contents of these dictionaries as they are when the function exits.

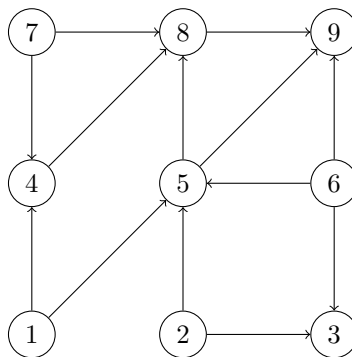
```
def bfs_shortest_paths(graph, source):
    status = {node: 'undiscovered' for node in graph.nodes}
    distance = {node: float('inf') for node in graph.nodes}
    predecessor = {node: None for node in graph.nodes}
    status[source] = 'pending'
    distance[source] = 0
    pending = deque([source])
    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v] = 'pending'
                distance[v] = distance[u] + 1
                predecessor[v] = u
                # append to right
                pending.append(v)
        status[u] = 'visited'
    return predecessor, distance
```



b) Mark the BFS trees produced on executing BFS on this graph.

**Problem 2.**

Consider the following directed graph.



a) Run Full\_DFS on the graph above. Make a bold arrow from node  $u$  to node  $v$  if  $u$  is the predecessor of node  $v$  in DFS. Use the convention that a node's neighbors are processed in ascending order by label.



b) Fill in the table below so that it contains the start and finish times of each node after a Full\_DFS is performed on the above graph. Assume node 1 as the source for the first DFS call. Begin your start times with 1.

Node	Start	Finish
1	<input type="text"/>	<input type="text"/>
2	<input type="text"/>	<input type="text"/>
3	<input type="text"/>	<input type="text"/>
4	<input type="text"/>	<input type="text"/>
5	<input type="text"/>	<input type="text"/>
6	<input type="text"/>	<input type="text"/>
7	<input type="text"/>	<input type="text"/>
8	<input type="text"/>	<input type="text"/>
9	<input type="text"/>	<input type="text"/>

c) Topologically sort the vertices of the graph.

**Problem 3.**

State whether the following statements are true or false.

- a) Full breadth first search on a directed graph always produces same number of BFS trees irrespective of order in which vertices are given and the neighbouring nodes are visited.

- b) Full breadth first search on an undirected graph always produces same number of BFS trees irrespective of order in which vertices are given and the neighbouring nodes are visited.

- c) Both Full BFS and Full DFS require atleast  $\Omega(V)$  memory.

- d) Consider a graph  $G$  on which BFS is run with node  $s$  as the source. Assume that BFS visits a node  $u$  in the graph before node  $v$ . Then  $d(s, u) < d(s, v)$

- e) Every directed acyclic graph has exactly one topological ordering.

**Problem 4.**

Given an undirected graph  $G=(V,E)$ , give an algorithm to find if the graph is disconnected.

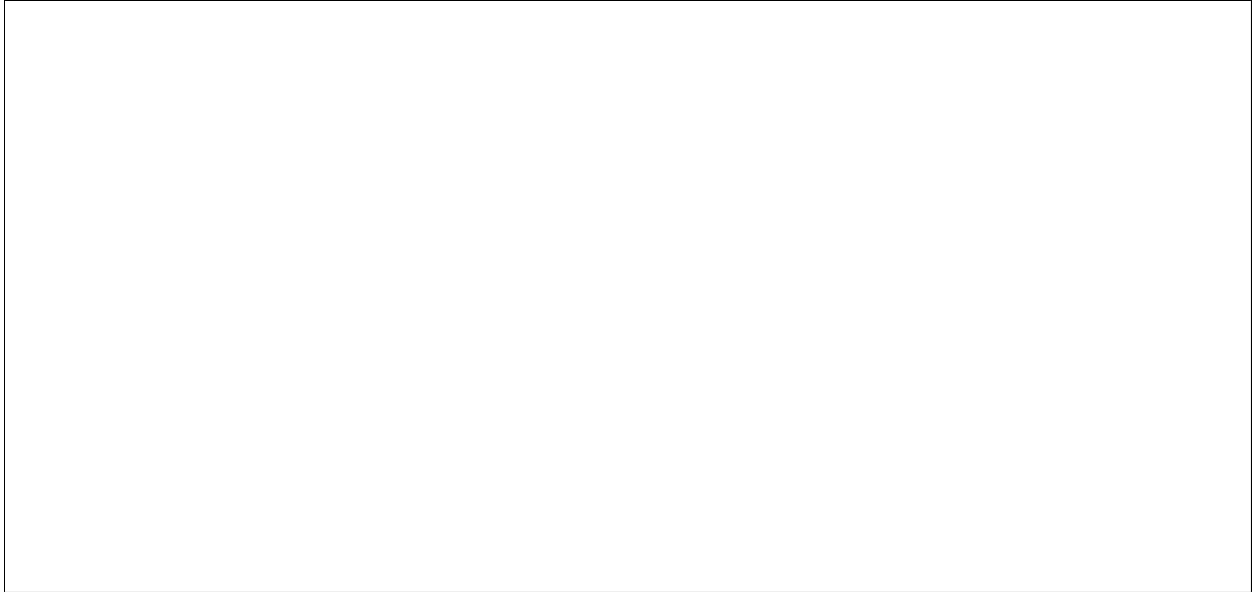


Figure 1: "Full" DFS

```
from data classes import dataclass
@dataclass
class Times:
    clock: int
    start: dict
    finish: dict

def full_dfs_times(graph):
    status = {node: 'undiscovered' for node in graph.nodes}
    predecessor = {node: None for node in graph.nodes}
    times = Times(clock=0, start={}, finish={})
    for u in graph.nodes:
        if status[u] == 'undiscovered':
            dfs_times(graph, u, status, times)
    return times, predecessor

def dfs_times(graph, u, status, predecessor, times):
    times.clock += 1
    times.start[u] = times.clock
    status[u] = 'pending'
    for v in graph.neighbors(u):
        # explore edge (u, v)
        if status[v] == 'undiscovered':
            predecessor[v] = u
            dfs_times(graph, v, status, times)
    status[u] = 'visited'
    times.clock += 1
    times.finish[u] = times.clock
```